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The importance of variable order when constraining correlation patterns between random parameters

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Constraining correlation patterns between random parameters

Situation:

RPL model allowing for correlation between *some* parameters

Restrict correlation between some parameters to 0

In practice, constrain elements of Cholesky matrix (instead of VC matrix)

Problem:

Cholesky factorisation is not invariant to the order of variables

→ Pattern of (un-)constrained elements of Cholesky matrix does not translate through to VC matrix

Example 1

Five variables, ordered as $v1$, $v2$, $v3$, $v4$ and $v5$.

Correlation between $v1$, $v3$ and $v5$, and $v2$, $v4$ and $v5$.

Cholesky matrix:

$$C_1 = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

Variance-covariance matrix:

$$VC_1 = \begin{bmatrix} {a_{11}}^2 & 0 & a_{31}a_{11} & 0 & a_{51}a_{11} \\ 0 & {a_{22}}^2 & 0 & a_{22}a_{42} & a_{22}a_{52} \\ a_{31}a_{11} & 0 & {a_{31}}^2 + {a_{33}}^2 & 0 & a_{51}a_{31} + a_{53}a_{33} \\ 0 & a_{42}a_{22} & 0 & {a_{42}}^2 + {a_{44}}^2 & a_{52}a_{42} + a_{54}a_{44} \\ a_{51}a_{11} & a_{52}a_{22} & a_{51}a_{31} + a_{53}a_{33} & a_{52}a_{42} + a_{54}a_{44} & {a_{51}}^2 + {a_{52}}^2 + {a_{53}}^2 + {a_{54}}^2 + {a_{55}}^2 \end{bmatrix}$$

Example 2

Same five variables, ordered $v5$, $v1$, $v2$, $v3$ and $v4$.

Same correlation pattern between $v1$, $v3$ and $v5$, and $v2$, $v4$ and $v5$.

Cholesky matrix:

$$C_2 = \begin{bmatrix} a_{55} & 0 & 0 & 0 & 0 \\ a_{15} & a_{11} & 0 & 0 & 0 \\ a_{25} & 0 & a_{22} & 0 & 0 \\ a_{35} & a_{31} & 0 & a_{33} & 0 \\ a_{45} & 0 & a_{42} & 0 & a_{44} \end{bmatrix}$$

Variance-covariance matrix:

$$VC_2 = \begin{bmatrix} {a_{55}}^2 & a_{15}a_{55} & a_{25}a_{55} & a_{35}a_{55} & a_{45}a_{55} \\ a_{15}a_{55} & {a_{15}}^2 + {a_{11}}^2 & a_{25}a_{15} & a_{35}a_{15} + a_{31}a_{11} & a_{45}a_{15} \\ a_{25}a_{55} & a_{25}a_{15} & {a_{25}}^2 + {a_{22}}^2 & a_{35}a_{25} & a_{45}a_{25} + a_{54}a_{22} \\ a_{35}a_{55} & a_{35}a_{15} + a_{31}a_{11} & a_{35}a_{25} & {a_{35}}^2 + {a_{31}}^2 + {a_{33}}^2 & a_{45}a_{35} \\ a_{35}a_{55} & a_{45}a_{15} & a_{45}a_{25} + a_{42}a_{22} & a_{45}a_{35} & {a_{45}}^2 + {a_{42}}^2 + {a_{44}}^2 \end{bmatrix}$$

Questions

1. Is the sensitivity of the Cholesky factorisation to variable order generally a problem in applications?

   → If yes, how to deal with it?

2. How to constrain (some) elements of the VC matrix to zero?