

Replicability, simulation error and robustness to non-parametric treatment of preference heterogeneity in discrete choice models

Mikołaj Czajkowski
Mateusz Buczyński, Wiktor Budziński

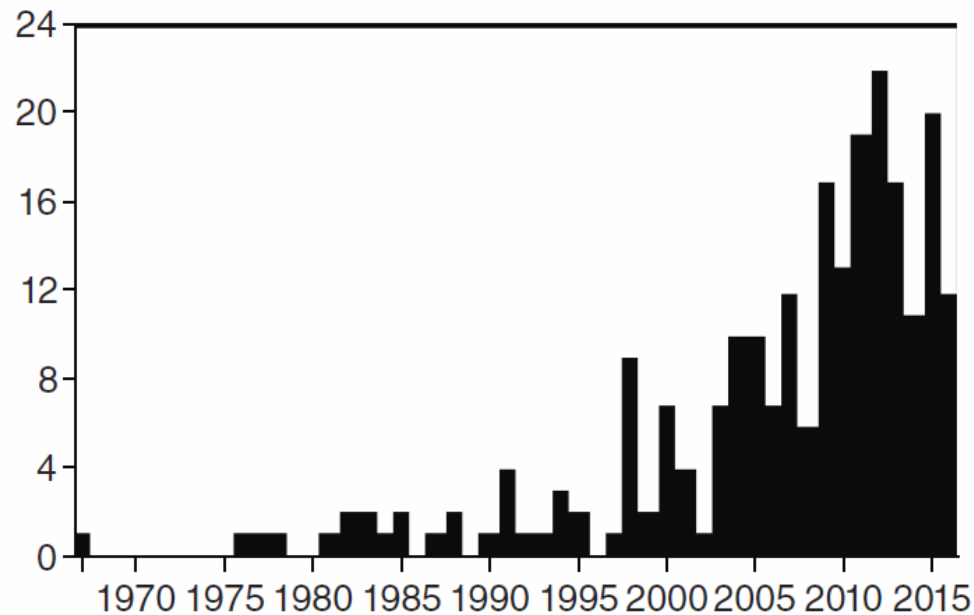
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Replication in economics

- Replication is crucial to credibility, confidence in findings, reliability ...
- Used to be quite rare in economics
 - Hampered by inaccessible data
 - A lot of effort
 - Low probability of good publication
 - Selection for unsuccessful replication
 - E.g., Leimer and Lesnoy (1982) vs. Feldstein (1974)
 - Mueller-Langer et al. (2017): less than 0.1% of 1,200 papers published in top journals (1974-2014) were replications

Trending reproducibility?

- Growing interest in replication
 - The rate may be low but it has been quickly rising (Duvendack et al., 2017)



Data Access Policies

- More and more journals require authors to share their data sets and codes for the sake of future replications
- DAP allows that only papers that have “clearly and precisely documented data available to any researcher for purposes of replication” would be accepted
 - McCullough and Vinod (2004) proven a high number of irreproducible studies submitted to the AER:

"We adduced copious evidence that solvers used by economists can produce inaccurate answers, gave examples of different packages giving different answers to the same nonlinear problems, and showed, at least in this journal, that researchers make no effort to verify the solutions from the solvers that they use. We believe this uncritical acceptance of solutions from nonlinear solvers to be a systemic problem in economic research."
 - and in the wake of the scandal AER has been following DAP since 2004
- Other top journals followed, some 51% 'currently' have at least soft or informal policy of data availability (Brown et al., 2014)
- Very strong DAP are mostly associated with top journals

DAP in environmental economics


- "Contemporary guidance ..." recommendations for stated preference methods (Johnston et al., 2017) are rather vague and weak in this respect:
 - *"Recommendation 23: All studies [...] should fully document study design, implementation, analyses, and results. Such transparency is crucial for the scientific credibility of studies and the appropriate interpretation and use of results. [...] Study reporting and archival documentation are important for many reasons. For example, ex post content validity assessment of a study and efforts to replicate study results require documentation of procedures and investigator decisions"*
 - No explicit requirement for making the data and software codes (and questionnaires) available for replication purposes
 - More concerned about sensitive information and protecting confidentiality of the participants

Making data available

- Some environmental economics journals implement DAP
 - 2013: Energy Economics
 - 2017: Journal of the Association of Environmental and Resource Economics
- Some 'encourage' sharing data and software codes
 - Journal of Environmental Economics and Management
 - Ecological Economics
 - Resource and Energy Economics
 - American Journal of Agricultural Economics
 - E.g., "Authors are encouraged to comply with all of this policy, but the editors would prefer partial compliance over non-compliance"
- While others do not even mention it
 - Environmental and Resource Economics
 - Review of Environmental Economics and Policy

Making software codes available

- The number of researchers who share their codes and estimation packages is growing
 - From "*I never share my codes*" to:



MONDAY 27 NOVEMBER 2017

CMC R Code now available

[SHARE](#)

The Choice Modelling Centre (CMC) at the University of Leeds has developed flexible estimation code for choice models in R. The code uses the complete opposite of a black-box approach, i.e. the user sees every step in the coding of a log-likelihood function. We believe this to be essential in ensuring a greater understanding by users of the very powerful models they have at their disposal.

Open source estimation packages and sharing software codes

– Other examples:

- Kenneth Train: <https://eml.berkeley.edu/~train/software.html>
- Chandra Bhat: http://www.caee.utexas.edu/prof/bhat/FULL_CODES.htm
- Michel Bierlaire: <http://biogeme.epfl.ch/>
- Sander van Cranenburgh: <https://www.advancedrrmmodels.com/>

– How I and my colleagues at UW deal with it:

- Matlab package for estimating various choice models
 - <https://github.com/czaj/>
- Make data sets, software codes, and various additional results available as an online supplement to each paper
 - <http://czaj.org/research/supplementary-materials>

Notable replication studies

- Camerer et al. (2016)
 - 18 AER and QJE papers
 - Significant effect in the same direction as in the original study for 11 replications (61%)
 - On average, the replicated effect size was 66% of the original
- Chang and Li (2018)
 - 67 papers published in 13 top journals
 - 65% of the authors supplied necessary files, while the rest either used confidential data or refused to send data
 - Successfully replicated 22 out of 67 (33%) papers without contacting the authors, expanding this set to 29 papers in case of assistance of original authors (43%)
- McCullough et al. (2006)
 - 266 papers from Journal of Money, Credit and Banking
 - 69 out of 186 archived data (others ignored the requirement), 58 out of 69 (84%) included data and code, 14 out of 62 (23%) could be replicated without contacting authors for additional instructions

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
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Special Issue on Replication in Energy Economics

Replication is important. It is perhaps not as important in economics as it is in medicine – where life and death can depend on the accuracy of research findings – but it is key to the credibility of our field and the confidence in our research findings. Yet, replication papers are rare in economics, probably because they take a lot of effort with a low probability of publication. Replication is also hampered by inaccessible data. In 2013, *Energy Economics* followed the example of the journals of the *American Economic Association* in demanding that data and code be accessible to the reader. Unfortunately, although we did publish a few papers (De Vita and Trachanas 2016, Pottier, Hourcade, and Espagne 2014),^[1] this step change in *replicability* did not lead to a step change in *replication*.

Energy Economics will therefore publish a special issue on replication. In this special issue, we will particularly welcome two types of papers, without excluding other forms of replication (Clemens 2015). **First, we would like to see replication of older but prominent research. Prominent papers would be ones that are frequently cited or used in policy making. This type of paper would ask whether the old results stand up if newer data are added and methods are brought up to date.** If not, why? At the core of this type of contribution would be a table with the original results, the best attempt at replication, and the results with additional data or alternative methods (Reed and Alm 2015). **Papers eligible for replication include all economics papers with some relation to energy.**

In recent years, *Energy Economics* has made data and code available for empirical papers. The second type of replication paper we would like to see takes a number of recent articles to check whether the results stand up if all the evidence is put together. For instance, different authors may have worked on the same data with different methods. Can the difference in findings be explained? Is there an objective way to distinguish between more and less credible results? In other cases, different authors may have used similar methods for different data, for example, for different countries, different economic sectors, or different energy sources. What happens to the results if the data are pooled? Again, the replication paper should revolve around a table with original results, replication results, and new results. As above, the replicated papers have to be in economics with some relation to energy. If the majority of replicated papers were published in Elsevier journals, then we will publish a *virtual special issue* consisting of the replicated paper(s), the replication paper, and the commentaries by the original authors.

Besides the special issue, we have created “replication paper” as a new type of submission. Replication will not end with this special issue.

Replication is key to research, but it can also be used to fight old battles or start new ones. Submissions will be policed for a polite and constructive contribution. Authors of the replicated papers will be invited to publish a commentary.

The set-up of our replication study

- I contacted the authors of the 15 top cited papers using discrete choice models applied in Energy Economics and asked for data

From: Mikołaj Czajkowski
Sent: Friday, January 13, 2017 3:25 AM
To: aa_goett@pacbell.net; katseye@seanet.com; train@econ.berkeley.edu
Subject: Request regarding your paper

Dear Andrew Goett, Kathleen Hudson and Kenneth Train,

I am writing a paper for the special issue of Energy Economics devoted to replication of the results of stated preference studies. The idea (blessed by Richard Tol, the editor-in-chief) is to use the 5-10 most cited papers dealing with energy economic issues and using stated preferences. In particular, I want to test if I can replicate the estimation results and additionally focus on whether the results change if:

- (1) one uses more precise simulation for the LL function (more draws, smart draws), if applicable,
- (2) one uses more flexible model specifications, e.g. MXL with all variables random and correlated (if the original model was not specified this way).

One of these papers is:

Goett, A. A., Hudson, K., and Train, K. E., 2000. Customers' Choice Among Retail Energy Suppliers: The Willingness-to-Pay for Service Attributes. The Energy Journal, 21(4):1-28.

I was wondering if you would be willing to make the data you used for this study available.

It is difficult to overestimate the value of replication in science. Whether the community of the authors of stated preference studies are willing to make their data available for replication is also an interesting experiment. I hope that the paper will be able to carry an optimistic message in this respect. Thank you very much in advance.

Best regards,
Mik

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Mikołaj Czajkowski
<http://czaj.org/>

	Published	Citations	Responded	Shared data
Achtnicht, M., 2012. German car buyers' willingness to pay to reduce CO2 emissions. <i>Climatic Change</i> , 113(3):679-697.	2012	138	4 days	1 month
Banfi, S., Farsi, M., Filippini, M., and Jakob, M., 2008. Willingness to pay for energy-saving measures in residential buildings. <i>Energy Economics</i> , 30(2):503-516.	2008	391	1 day	no (confidentiality)
Bergmann, A., Colombo, S., and Hanley, N., 2008. Rural versus urban preferences for renewable energy developments. <i>Ecological Economics</i> , 65(3):616-625.	2008	180	1 day	15 months / 2 days
Bergmann, A., Hanley, N., and Wright, R., 2006. Valuing the attributes of renewable energy investments. <i>Energy Policy</i> , 34(9):1004-1014.	2006	395	1 day	no (no longer available)
Carlsson, F., and Martinsson, P., 2008. Does it matter when a power outage occurs? — A choice experiment study on the willingness to pay to avoid power outages. <i>Energy Economics</i> , 30(3):1232-1245.	2008	125	5 days	7 months
Dimitropoulos, A., and Kontoleon, A., 2009. Assessing the determinants of local acceptability of wind-farm investment: A choice experiment in the Greek Aegean Islands. <i>Energy Policy</i> , 37(5):1842-1854.	2009	172	-	-
Goett, A. A., Hudson, K., and Train, K. E., 2000. Customers' Choice Among Retail Energy Suppliers: The Willingness-to-Pay for Service Attributes. <i>The Energy Journal</i> , 21(4):1-28.	2000	274	1 day	no (no longer available, offered a similar dataset)
Hidrue, M. K., Parsons, G. R., Kempton, W., and Gardner, M. P., 2011. Willingness to pay for electric vehicles and their attributes. <i>Resource and Energy Economics</i> , 33(3):686-705.	2011	462	-	-
Krueger, A. D., Parsons, G. R., and Firestone, J., 2011. Valuing the visual disamenity of offshore wind power projects at varying distances from the shore: an application on the Delaware shoreline. <i>Land Economics</i> , 87(2):268-283.	2011	94	-	-
Ladenburg, J., and Dubgaard, A., 2007. Willingness to pay for reduced visual disamenities from offshore wind farms in Denmark. <i>Energy Policy</i> , 35(8):4059-4071.	2007	187	-	
Mabit, S. L., and Fosgerau, M., 2011. Demand for alternative-fuel vehicles when registration taxes are high. <i>Transportation Research Part D: Transport and Environment</i> , 16(3):225-231.	2011	127	1 day	11 days
MacKerron, G. J., Egerton, C., Gaskell, C., Parpia, A., and Mourato, S., 2009. Willingness to pay for carbon offset certification and co-benefits among (high-)flying young adults in the UK. <i>Energy Policy</i> , 37(4):1372-1381.	2009	174	-	-
Potoglou, D., and Kanaroglou, P. S., 2007. Household demand and willingness to pay for clean vehicles. <i>Transportation Research Part D: Transport and Environment</i> , 12(4):264-274.	2007	344	1 year	no (asked to co-author the paper)
Scarpa, R., and Willis, K., 2010. Willingness-to-pay for renewable energy: Primary and discretionary choice of British households' for micro-generation technologies. <i>Energy Economics</i> , 32(1):129-136.	2010	317	1 day	3 days

Response summary

- Out of the authors of the 15 papers I contacted:
 - 9 (60%) responded
 - Nearly all within a week
 - Most on the same day
 - Possible bias – I know some of the authors personally
 - 5 (33%) provided data for replication purposes

What exactly is replication?

- Replication is a task which recreates the experiment with an unaltered formal specification, using identical population, but ...
 - Various classifications in Clemens (2017), Hunter (2001), Hamermesh (2007), Arulampalam et al., (1997), Hubbard and Vetter (1996), Pesaran et al. (2001)
 - Generally:

	Specification	Sample	Population
<i>Pure replication</i>			
<i>Verification</i>	same	same	same
<i>Replication of first degree</i>			
<i>Reproduction</i>	same	new	same
<i>Statistical replication</i>			
<i>Reanalysis</i>	new	same	same
<i>Extension</i>	same	new	new
<i>Scientific replication</i>	new	new	new

Our approach to replication

1. *Pure replication*: Can we use author-provided data sets and replicate their results?
 - Same data set
 - Ask the authors for clarifications, if necessary
 - Use the same specification of the model
 - Replicate the MNL model results (if available)
 - Replicate the MXL model results (within "machine precision")
 - Simulation error
2. *Reanalysis*: Can we use author-provided data sets and find a better specification for their model?
 - Improve the MXL model by using different random parameters distributions, allowing for correlations, looking for better convergence, etc.
 - Non-parametric approach to modelling preference heterogeneity
 - The Logit-Mixed Logit model (Train, 2017)

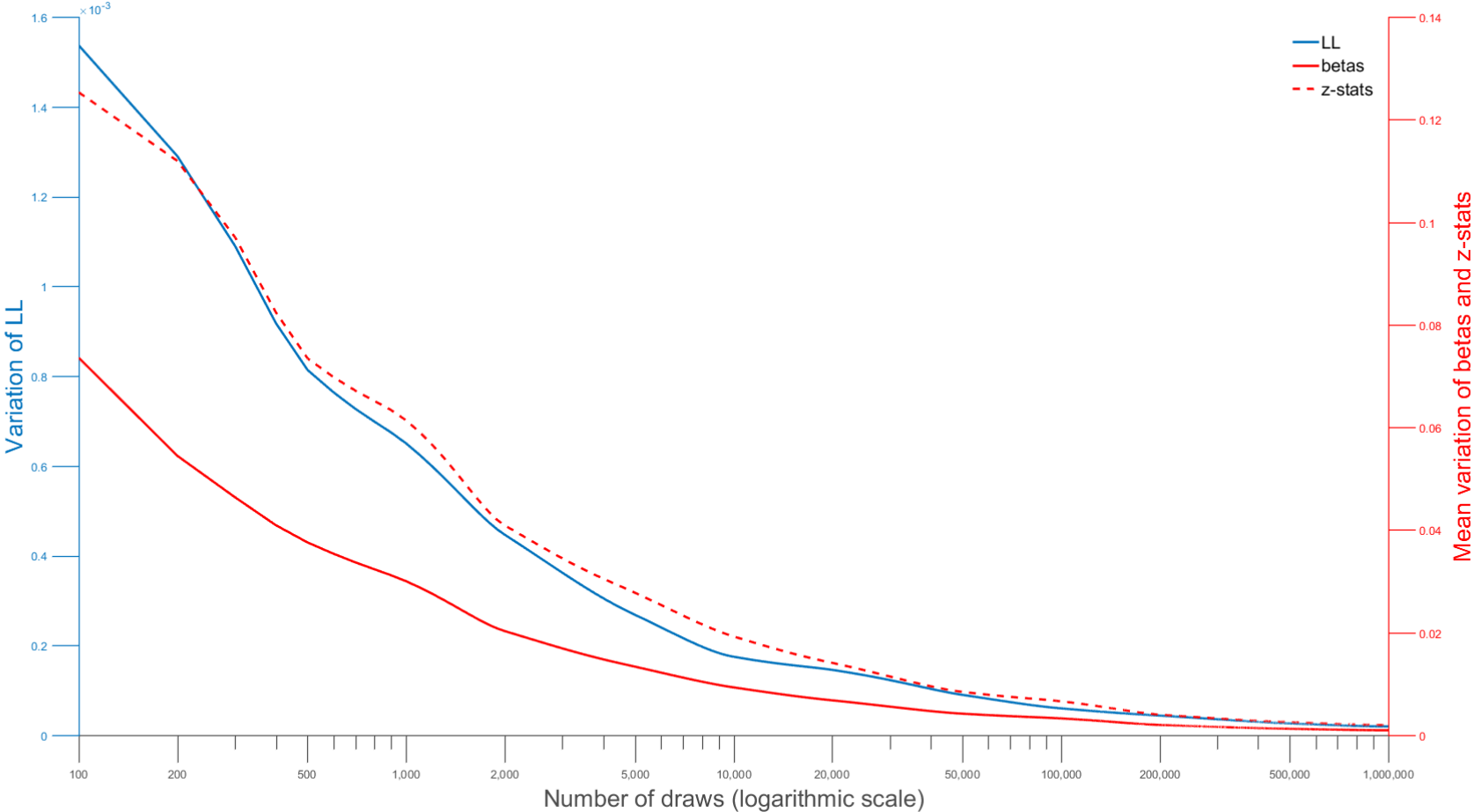
Replication of the MNL model results

	Needed additional contacts or clarifications	Possible to replicate sample	LL MNL original	LL MNL replicated	Mean absolute difference of B
Achtnicht (2012)	yes	yes	-6,095.39	-6,095.39	< 0.01%
Bergmann, Colombo, and Hanley (2008)	yes	?	n/a	-495.59	
Carlsson, and Martinsson (2008)	no	yes	-2,522.50*	-2,522.50	< 0.01%
Mabit and Fosgerau (2011)	yes	yes	n/a	-13,373.37	
Scarpa, and Willis (2010)	no	yes	-7,328.88	-7,350.75	< 0.01%

Replication of the MXL model results

- Mixed (random parameters) logit models estimated using the simulated maximum likelihood method
 - Necessarily associated with simulation error
 - Depends on the number and type of draws
 - A different set of draws = somewhat different estimation results
- How large is the simulation error?
 - Czajkowski, M., and Budziński, W., 2017. Simulation error in maximum likelihood estimation of discrete choice models. Paper presented at the 6'th International Choice Modelling Conference, Cape Town.

Simulation error vs. the number of draws



Simulation error – Design of the simulation study

Repetitions	Draws		Datasets		
	Types of draws	Number of draws	Number of choice tasks per individual	Number of individuals	Experimental designs
1,000	<i>pseudo-random</i> <i>MLHS</i> <i>Halton</i> <i>Sobol</i>	100			
		200			
		500			
		1,000			
		2,000			
		5,000	4	400	OOD-design
		10,000	8	800	MNL-design
		20,000*	12	1,200	MXL-design
		50,000*			
		100,000*			
		200,000*			
		500,000*			
		1,000,000*			

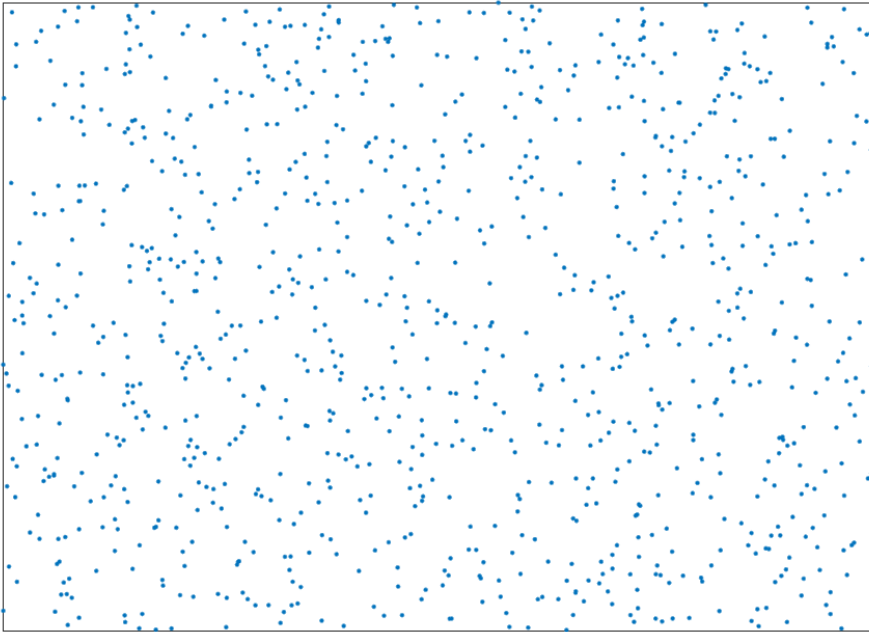
*Selected settings only.

Simulation error – Design of the simulation study

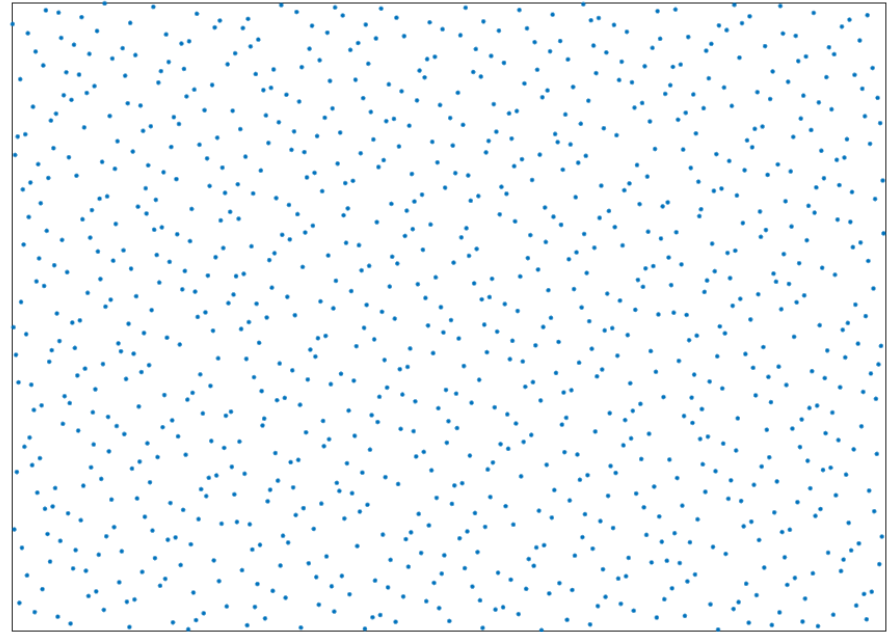
Explanatory variables (choice attributes)	Assumed parameter distribution	Possible values of the explanatory variables		
		Alternative 1 (status quo / opt-out)	Alternative 2	Alternative 3
X_1 (alternative specific constant)	$N(-1.0, 0.5)$	$X_1 = 1$	$X_1 = 0$	$X_1 = 0$
X_2 (dummy)	$N(1.0, 0.5)$	$X_2 = 0$	$X_2 \in \{0, 1\}$	$X_2 \in \{0, 1\}$
X_3 (dummy)	$N(1.0, 0.5)$	$X_3 = 0$	$X_3 \in \{0, 1\}$	$X_3 \in \{0, 1\}$
X_4 (dummy)	$N(1.0, 0.5)$	$X_4 = 0$	$X_4 \in \{0, 1\}$	$X_4 \in \{0, 1\}$
X_5 (discrete)	$N(-1.0, 0.5)$	$X_5 = 0$	$X_5 \in \{1, 2, 3, 4\}$	$X_5 \in \{1, 2, 3, 4\}$

Pseudo-random vs. Halton sequence

Scatter plot of 1000 draws for 2 pseudo-random sequences

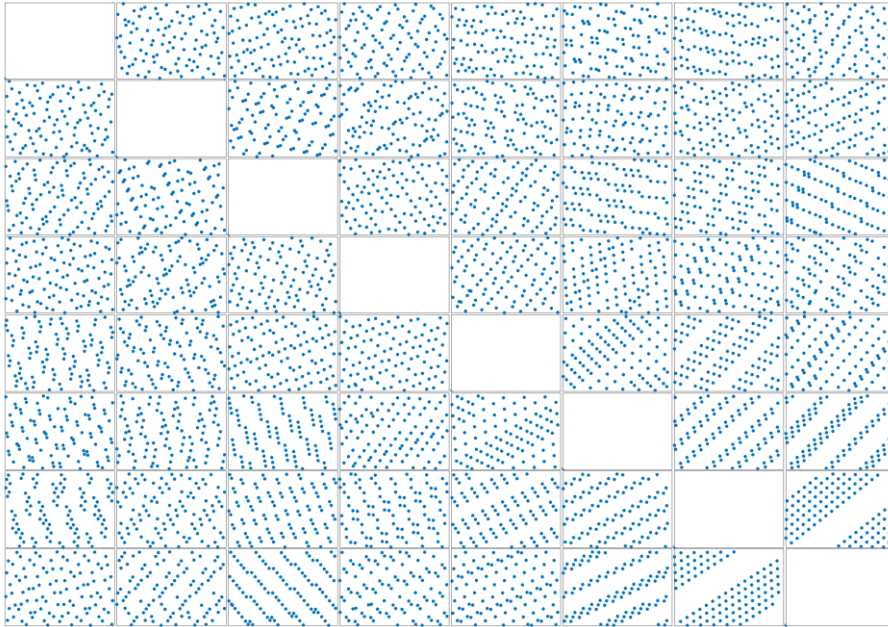


Scatter plot of 1000 draws for 2 Halton sequences

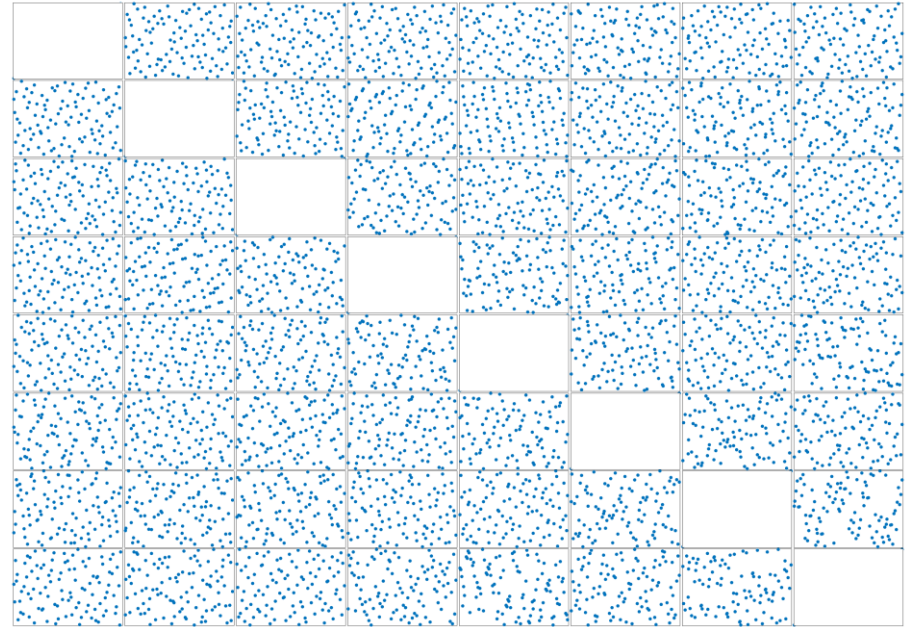


Halton vs. scrambled Halton sequence

Scatter plot matrix of 100 draws for 8 Halton sequences



Scatter plot matrix of 100 draws for 8 scrambled Halton sequences



Simulation error – Methodology of comparisons

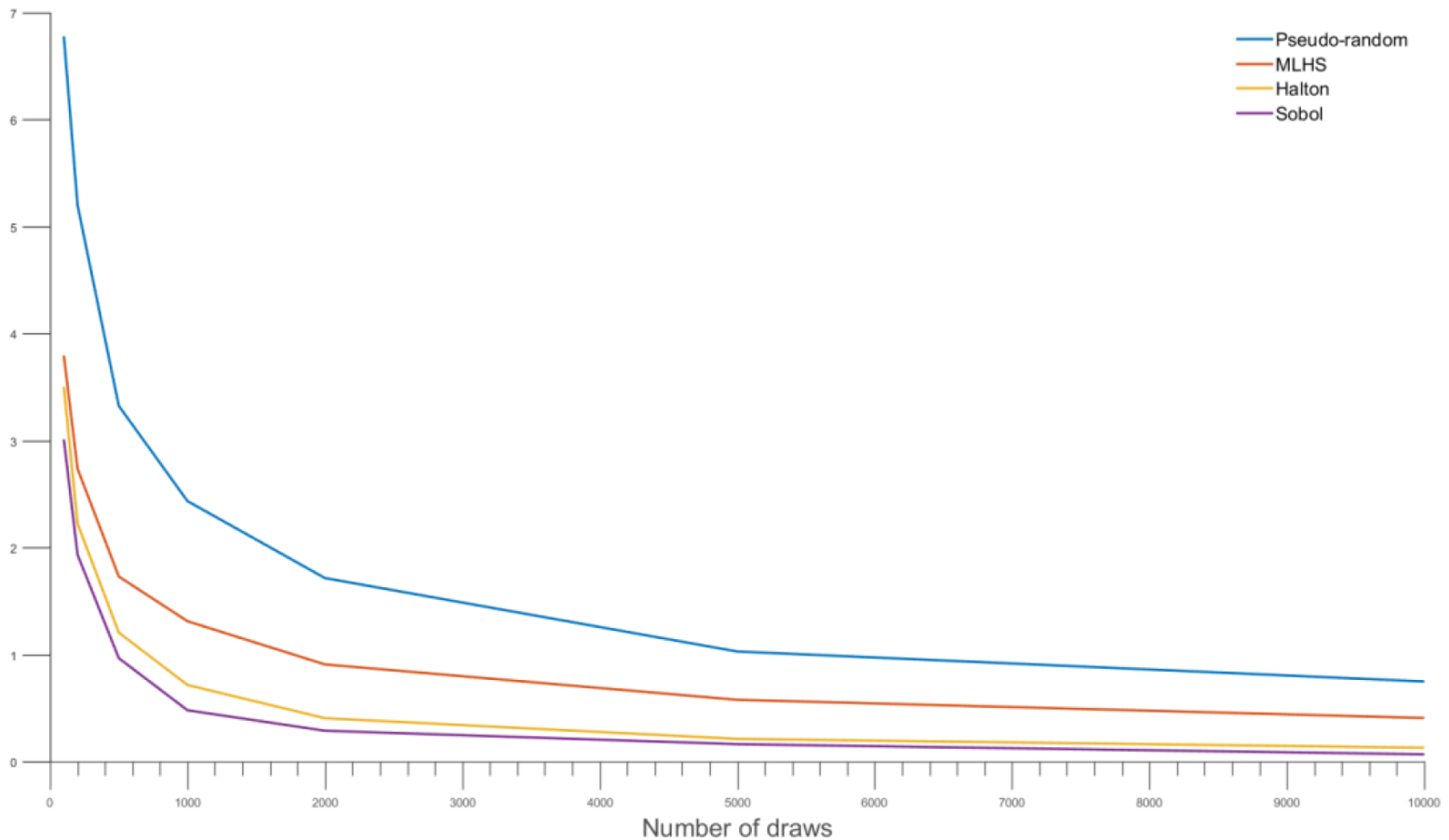
- We need a measure that takes expected values into account but also penalizes for variance
 - For typical equality tests – the larger the variance, the more difficult to reject the equality hypothesis
- Testing equivalence instead of equality
 - Reverse the null and the alternative hypotheses
 - Test if the absolute difference is higher than a priori defined ‘acceptable’ level
- Minimum Tolerance Level (MTL)
 - What is the minimum ‘acceptable’ difference that allows to conclude that two values are equivalent at the required significance level
 - How many draws of type A are required, so that with 95% probability the difference in LL / estimates / s.e. / z-stats is not going to be statistically different than:
 - The critical value of the LR-test
 - If the model was estimated using n draws of type B

Example – using MTL for the values of the LL function

- Re-estimating the model using a different set of draws is likely to result in a somewhat different value of the LL function
- If LL is used for inference (e.g., LR-test), it is possible to conclude that one specification is superior to another (equally good) specification only because one was more ‘lucky’ with the draws
- By using the MTL approach we are able to evaluate the probability of such an outcome
 - Assume $\alpha = 0.05$, the interpretation of $MTL_{0.05}$ is that with 95% probability using a different set of draws would not cause the difference in LL values to be higher than $MTL_{0.05}$
 - We can provide recommendations for the minimum number of draws that would result in $MTL_{0.05}$ lower than the specified level
 - E.g., the critical value of the LR-test – probability of erroneously concluding that one model is preferred to another (because of simulation error) is lower than a desired significance level, e.g., 0.05

Simulation error – Results: relative performance of types of draws

– Example: $MTL_{0.05}$ of LL for MXL-design, 400 x 4:



Simulation error – Results: Sobol draws consistently perform best

– Percent of additional draws needed to achieve the same simulation error as Sobol draws:

	<i>Pseudo-random</i>	<i>MLHS</i>	<i>Halton</i>
LL	889% [776% - 1,020%]	305% [258% - 360%]	66% [47% - 87%]
Parameter estimates	361% [331% - 392%]	209% [189% - 232%]	48% [38% - 58%]
z-stats	347% [321% - 375%]	200% [182% - 219%]	51% [42% - 60%]

* Based on regression analysis

Simulation error –

Results: how many draws are 'enough'?

- Using more draws is always better to using fewer draws
- How many are 'enough' depends on the desired precision level
- Log-likelihood:
 - Imagine you are comparing 2 specifications using LR-test (d.f. = 1)
 - Simulation error low enough to have 95% probability of not erroneously concluding that one model is better than the other
 - In other words, 95% of the times the (simulation driven) difference in LL must be lower than 1.9207 (at $\alpha = 0.05$)

	400 x 4	800 x 4	1,200 x 4	400 x 8	800 x 8	1,200 x 8	400 x 12	800 x 12	1,200 x 12
$p = 0.05$	120	230	340	300	600	890	470	920	1,370
$p = 0.01$	300	575	850	750	1,500	2,225	1,175	2,300	3,425

Simulation error –

Results: how many draws are ‘enough’?

– Parameter estimates:

- No absolute difference level
- The numbers of draws required for 95% probability that the difference between parameter estimates :

	400 x 4	800 x 4	1,200 x 4	400 x 8	800 x 8	1,200 x 8	400 x 12	800 x 12	1,200 x 12
< 5%	2,050	1,220	870	890	530	380	1,130	670	480
< 1%	33,420	19,850	14,180	14,450	8,590	6,130	18,450	10,960	7,820

- More draws required for standard deviations, ASC, dummies, fewer required for means, cost
- Similar results for comparisons with models estimated using 1,000,000 draws

Simulation error – Results: how many draws are ‘enough’?

– Overall (LL and parameter estimates):

	400 x 4	800 x 4	1,200 x 4	400 x 8	800 x 8	1,200 x 8	400 x 12	800 x 12	1,200 x 12	average
5%	2,050	1,220	870	890	600	890	1,130	920	1,370	1,000
1%	33,420	19,850	14,180	14,450	8,590	6,130	18,450	10,960	7,820	15,000

– Generally, advisable to check how many draws are necessary for parameter estimates and LL to stabilize

“It must take ages to estimate models with so many draws!”

- Estimation time (1 iteration = LL function evaluation + gradient)
 - Data set: 400 respondents x 4 choice tasks
 - CPU: Intel E5-2687W @ 3.00 GHz (12-core), no GPU used
 - Efficient implementation (Matlab, <https://github.com/czaj/dce>)

Number of draws	1,000	10,000	100,000	1,000,000
Iteration time	0.2 s	1 s	10 s	100 s

How fast and accurate are common statistical packages?

Software	Optimization method	Cores used	Estimation time (hh:mm:ss)	LL
NLOGIT 5 (unshuffled H)	BFGS	1	3:09:33	-5789.0590
NLOGIT 5 (shuffled H)	BFGS	1	2:25:00	-5937.0583
Stata MP 14 (H)		4	5:54:06	-5789.1129
PythonBiogeme	TR	10	1:07:29	-5795.2210
PythonBiogeme	CFSQP	10	1:36:44	-5790.8960
PythonBiogeme	SOLVOPT	10	2:48:42	-5793.6980
R (CMC Leeds)		1	5:49:08	-5789.0351
R (Erlend Sandorf)	BFGS	10	0:50:30	-5788.9905
R (Erlend Sandorf)	BHHH	10	0:37:42	-5789.0875
Matlab	QN	10	0:03:59	-5789.0446
Matlab	TR	10	0:00:33	-5789.0386

800 respondents x 8 choice tasks, 3 alternatives, 5 attributes, 10,000 scrambled Sobol* draws, convergence tolerance level: 10^{-8} , starting values: **0**, CPU: Intel i7-7900X @ 3.30 GHz (10-core)
 95% c.i. for LL: Sobol (-5788.93 ; -5789.20), Halton (-5788.66 ; -5789.46)

Replication of the MXL model results

- MXL models => we expect some variation of the results due to simulation bias

	LL MXL original	LL MXL replicated	Mean absolute difference of B
Achtnicht (2012)	-5,284.61	-5,280.18	3.93%
Bergmann et al. (2008)	-470.30	-481.89	14.73%
Carlsson et al. (2008)	-2,471.44	-2,471.99	9.37%
Mabit et al. (2011)	-11,758.00	-11,750.08	1.29%
Scarpa et al. (2010)	-5,744.01	-5,729.41	6.36%

Results of the replication exercise – summary

- 33% of contacted authors provided data for replication
- MNL results generally replicated well
- MXL results – substantial deviations in LL and parameter estimates
 - Large simulation error
 - Convergence problems
 - Other?
- Findings parallel to those of McCullough and Vinod (2004) for AER
 - Non-linear models requiring simulation require more care to produce reliable and precise results than generally believed
 - Possible remedies
 - Awareness
 - Best-practice optimization techniques
 - Making datasets, codes and documentation available – transparency and replication
 - Only publish results you are sure of

How about robustness to reanalysis?

- Are there better specifications of the authors' models available?
 1. A better specification for the MXL model?
 2. The Logit-Mixed Logit model (Train, 2016) –state-of-the-art non-parametric approach to modelling preference heterogeneity

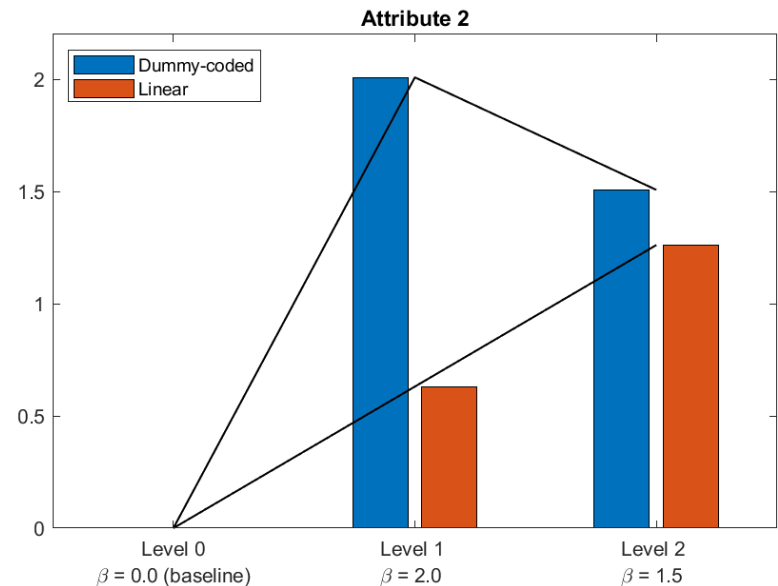
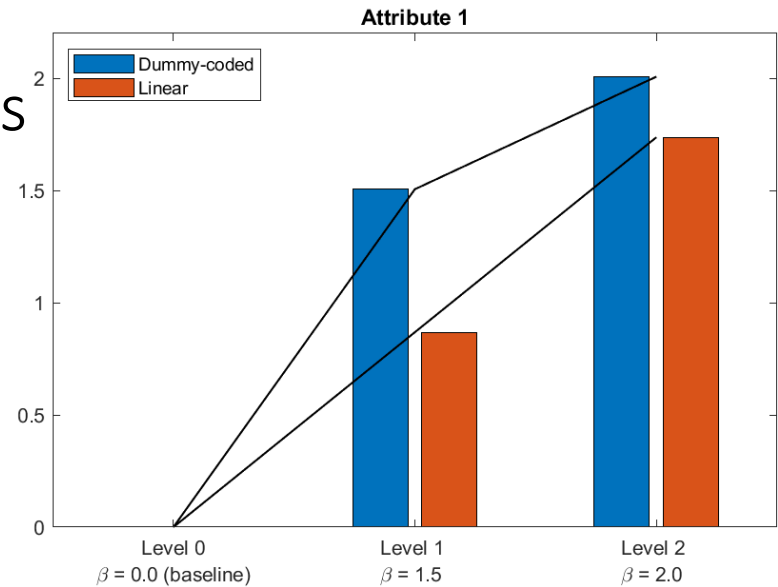
- 1. A better specification for the MXL model?
 - MXL has been around for some 20 years now
 - Theoretically, MXL can approximate any choice model to any degree of accuracy McFadden and Train (2000)
 - We have learned a lot about how to use it to get closer to the truth that can be learned from data

- Let's try some typical specifications people use these days (state-of-practice@2018), avoiding unjustified restrictions (examples follow), and keeping things simple
 - Make all parameters random, uncorrelated or correlated
 - Try different distributions (normal and log-normal only)
 - Keep variable transformations, drop socio-demographic covariates

Example #1

Do not force non-linear effects into linear coding

	DGP	MNL non-linear	MNL linear
θ_{11}	1.5	1.5058 (0.1436)	0.8687*** (0.0657)
θ_{12}	2	2.0074 (0.1499)	0.6308*** (0.0609)
θ_{21}	2	2.0088 (0.1491)	1.7373** (0.1314)
θ_{22}	1.5	1.5073 (0.1433)	1.2616* (0.1219)
θ_3	1	1.0053 (0.0570)	0.8976** (0.0506)
LL		-648.10 (22.3343)	-726.19 (21.4243)
AIC/n		1.3062 (0.0447)	1.4584 (0.0428)
BIC/n		0.6173 (0.0138)	0.5688 (0.0134)



=> Parameter estimates biased

Example #2

Do not use effects coding

– Bech and Gyrd-Hansen (2010) setting

	DGP	MNL dummy	MNL effects	MNL effects*
β_{ASC}	-1.114	-1.1159*** (0.0887)	-1.1159*** (0.0887)	-0.0146 (0.0798)
β_{nurse}	2.20	2.2026*** (0.0783)	1.1013*** (0.0391)	1.1013*** (0.0391)
β_{visits}	0.25	0.2505*** (0.0187)	0.2505*** (0.0187)	0.2505*** (0.0187)
β_{cost}	-1	-1.0011*** (0.0305)	-1.0011*** (0.0305)	-1.0011*** (0.0305)
LL		-2556.8741*** (42.2159)	-2556.8741*** (42.2159)	-2556.8741*** (42.2159)
AIC/n		0.8536*** (0.0141)	0.8536*** (0.0141)	0.8536*** (0.0141)
BIC/n		0.8581*** (0.0141)	0.8581*** (0.0141)	0.8581*** (0.0141)

⇒ Effects coding does not influence ASC parameter (all it does is making interpretation more complicated)

⇒ Only if SQ alternative levels incorrectly coded as 0 rather than -1 (non-conditional model) ASC parameter changes (c.f. Bech and Gyrd-Hansen, 2010)

Example #3

Why constrain some parameters to be non-random?

	DGP	MNL	MXL_d
θ_{ASC_SQ}	-1	0.5834*** (0.0907)	-0.9759*** (0.2019)
θ_{11}	1.5	1.3184*** (0.0496)	1.5051*** (0.0596)
θ_{12}	2	1.7505*** (0.0450)	2.0100*** (0.0578)
θ_2	-1	-0.8872*** (0.0899)	-1.0181*** (0.0929)
σ_{ASC-SQ}	3		3.0241*** (0.1621)
σ_{11}	0		0.0706 (0.1250)
σ_{12}	0		0.1190 (0.1770)
σ_2	0		0.1711 (0.2265)
LL		-5387.8391 (55.1879)	-4713.9141 (43.7439)
AIC/n		0.4301 (0.0045)	0.4755 (0.0036)
BIC/n		1.7973 (0.0184)	1.5740 (0.0146)

Alternatives correlated
 ⇒ Parameter estimates biased
 (ASC parameter estimate
 can even be of wrong sign)

Example #4

Dummy coded attributes are likely correlated

	DGP	MXL_d dummy-coded	MXL	MXL_d linear
θ_{11}	1.5	1.1688*** (0.0059)	1.4093*** (0.0074)	0.8988*** (0.0041)
θ_{12}	2	1.6386*** (0.0065)	1.8872*** (0.0068)	
θ_2	1	0.8545*** (0.0063)	0.9203*** (0.0074)	0.8854*** (0.0069)
σ_{11}	1.2247	0.5381*** (0.0012)	1.2064*** (0.0001)	0.6517*** (0.0011)
$\sigma_{11_{12}}$	1.2247		1.1635*** (0.0004)	
σ_{12}	1.4142	0.7617*** (0.0042)	0.5578*** (0.0014)	
σ_2	1	0.8601*** (0.0047)	0.9100*** (0.0057)	0.8853*** (0.0059)
LL		-4806.8794 (9.4072)	-4735.1182 (9.3574)	-4763.6841 (9.3434)
AIC/n		0.4694 (0.0008)	0.4751 (0.0008)	0.4732 (0.0008)
BIC/n		1.6056 (0.0031)	1.5850 (0.0031)	1.5899 (0.0031)

Not accounting for correlations
 \Rightarrow Parameter estimates biased

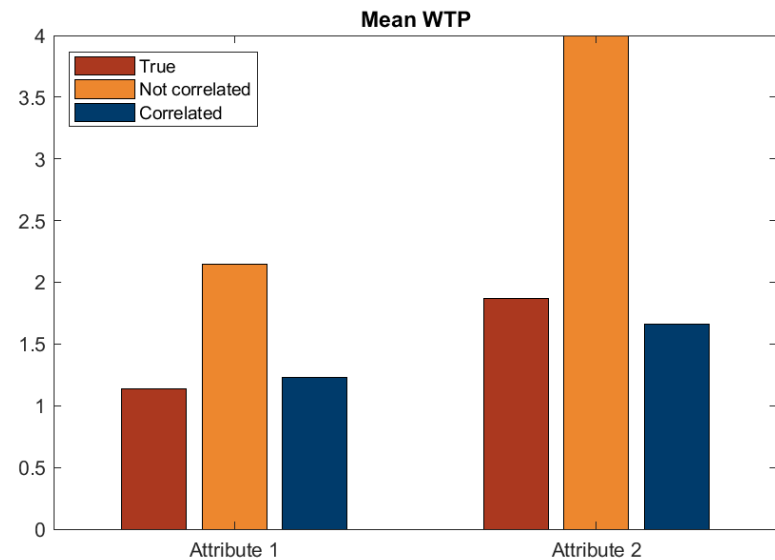
(linear coding can even
 outperform dummy coding, as
 linear implies 100% correlation)

Example #5

Allowing for correlations can account for scale heterogeneity (to some extent)

- Here – all parameters log-normally distributed, uncorrelated, scale random, log-normally distributed => allowing for correlations results in no bias in WTP

	DGP	MXL_d	MXL
WTP_1	1.1331	2.1438	1.2301
WTP_2	1.8682	4.2501	1.6600
LL		-11853.24 (8.5899)	-11830.07 (8.5364)
AIC/n		0.3823 (0.0003)	0.3834 (0.0003)
BIC/n		1.9765 (0.0014)	1.9732 (0.0014)



Not accounting for scale heterogeneity
=> Parameter estimates biased
allowing for correlations will reduce this bias

Are there better specifications of the authors' models available?

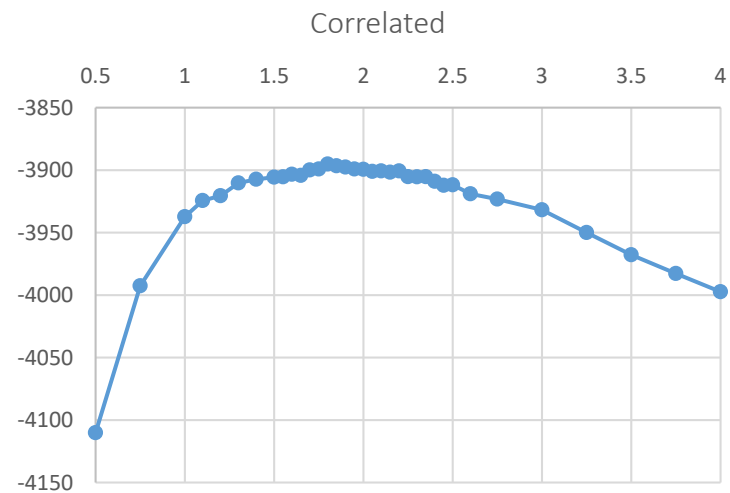
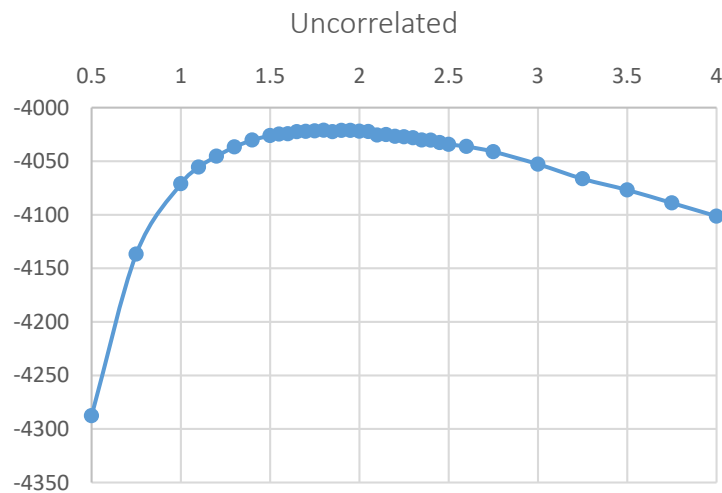
2. Non-parametric approach to modelling preference heterogeneity
 - the Logit-Mixed Logit model (Train, 2016)
 - Let mixing distribution of β be discrete, with sufficiently large and dense finite support set
 - Arbitrarily assume parameter range for each variable (e.g., $\beta_1 \in [-5, 5]$)
 - Take S evenly spaced points from within the parameter space
 - For each value of β , the choice probability is given by the logit formula
 - Let the probability that individual i 's utility function coefficient is β_r be given by another logit formula
$$P(\beta_i = \beta_r) = \frac{\exp(\alpha z(\beta_r))}{\sum_{s \in S} \exp(\alpha z(\beta_s))}$$
 - z are functions chosen to capture the shape of the probability mass function
 - Polynomials
 - Step functions
 - Splines (of a required order)
 - This specification gives a lot of flexibility and can easily approximate various distributions to a desired precision
 - Estimation is fast, as all logit-based choice probabilities can be calculated before the actual optimization (with respect to α) is done

The LML model – modelling the probability mass functions \mathbf{z}

- Probability mass functions (\mathbf{z}) tried:
 - Approximately normal (AN)
 - Legendre polynomial (Poly)
 - Step function (Step)
 - Linear spline (Spline L)
 - Cubic spline (Spline C)
 - Piece-wise cubic spline (Spline PW-C)
 - Piece-wise cubic Hermite interpolating spline (Spline - PW-CHI)
- Order of the probability mass functions (\mathbf{z}) considered: 2-10
 - The order of a polynomial (AN, Poly)
 - The number of step-function segments (excluding the reference segment)
 - The number of knots of splines (excluding the boundary knots)
- Relatively poor convergence
 - We used MSL with 1,000 grid points x 1,000 Sobol draws for probability evaluations
 - We used 10 partly-random starting values for each specification

The LML model – how to choose the support set for each parameter?

- Train (2016) uses MXL-based: mean $\pm 2 \cdot$ s.d.
- Sensitivity of the results to the range around the mean
 - Average across orders of 2,...,10 and the 7 z functions considered



- The LML seems quite sensitive to the selection of parameter space
- 1.8 was the best here, but many non-MXL-based possibilities available

Achtnich (2012)

Order	Dist	FullCov	LL	param.	AIC/n	BIC/n	Order	Dist	FullCov	LL	param.	AIC/n	BIC/n
MNL	replicated		-6095.39	15	3.4060	3.4319							
MNL			-6115.61	11	3.4151	3.4340							
MXL_d	replicated	0	-5280.18	25	2.9572	3.0003	MXL	replicated	1	-5107.91	70	2.8862	3.0069
MXL_d	n	0	-5332.96	22	2.9849	3.0229	MXL	n	1	-5166.28	77	2.9227	3.0554
MXL_d	n + l	0	-5274.34	22	2.9522	2.9902	MXL	n + l	1	-5066.48	77	2.8670	2.9998
2	AN	0	-5376.59	22	3.0092	3.0472	2	AN	1	-5265.42	77	2.9779	3.1107
2	Poly	0	-5376.59	22	3.0092	3.0472	2	Poly	1	-5265.42	77	2.9779	3.1107
2	Step	0	-5396.14	11	3.0140	3.0330	2	Step	1	-5345.97	66	3.0167	3.1305
2	Spline L	0	-5367.68	33	3.0104	3.0673	2	Spline L	1	-5258.81	88	2.9804	3.1321
2	Spline C	0	-5367.37	33	3.0102	3.0671	2	Spline C	1	-5267.12	88	2.9850	3.1367
2	Spline PW-C	0	-5362.78	33	3.0077	3.0646	2	Spline PW-C	1	-5283.65	88	2.9942	3.1459
2	Spline PW-CHI	0	-5368.87	33	3.0111	3.0680	2	Spline PW-CHI	1	-5270.86	88	2.9871	3.1388
3	AN	0	-5363.64	33	3.0082	3.0650	3	AN	1	-5253.26	88	2.9773	3.1290
3	Poly	0	-5363.64	33	3.0082	3.0650	3	Poly	1	-5253.26	88	2.9773	3.1290
3	Step	0	-5371.85	22	3.0066	3.0445	3	Step	1	-5304.27	77	2.9996	3.1323
3	Spline L	0	-5348.07	44	3.0056	3.0815	3	Spline L	1	-5244.46	99	2.9785	3.1492
3	Spline C	0	-5346.81	44	3.0049	3.0808	3	Spline C	1	-5251.60	99	2.9825	3.1532
3	Spline PW-C	0	-5343.41	44	3.0030	3.0789	3	Spline PW-C	1	-5256.36	99	2.9851	3.1558
3	Spline PW-CHI	0	-5349.70	44	3.0065	3.0824	3	Spline PW-CHI	1	-5246.43	99	2.9796	3.1503
4	AN	0	-5351.24	44	3.0074	3.0832	4	AN	1	-5239.19	99	2.9756	3.1462
4	Poly	0	-5351.24	44	3.0074	3.0832	4	Poly	1	-5239.19	99	2.9756	3.1462
4	Step	0	-5367.82	33	3.0105	3.0674	4	Step	1	-5299.08	88	3.0028	3.1545
4	Spline L	0	-5335.09	55	3.0045	3.0993	4	Spline L	1	-5229.53	110	2.9763	3.1660
4	Spline C	0	-5333.40	55	3.0036	3.0984	4	Spline C	1	-5239.52	110	2.9819	3.1715
4	Spline PW-C	0	-5332.80	55	3.0032	3.0980	4	Spline PW-C	1	-5257.28	110	2.9918	3.1814
4	Spline PW-CHI	0	-5336.77	55	3.0054	3.1003	4	Spline PW-CHI	1	-5231.85	110	2.9776	3.1672
5	AN	0	-5337.84	55	3.0060	3.1009	5	AN	1	-5226.50	110	2.9746	3.1643
5	Poly	0	-5337.84	55	3.0060	3.1009	5	Poly	1	-5226.48	110	2.9746	3.1643
5	Step	0	-5356.58	44	3.0104	3.0862	5	Step	1	-5276.13	99	2.9962	3.1668
5	Spline L	0	-5327.35	66	3.0063	3.1201	5	Spline L	1	-5225.32	121	2.9801	3.1887
5	Spline C	0	-5322.75	66	3.0038	3.1175	5	Spline C	1	-5230.25	121	2.9829	3.1915
5	Spline PW-C	0	-5322.81	66	3.0038	3.1176	5	Spline PW-C	1	-5242.02	121	2.9894	3.1980
5	Spline PW-CHI	0	-5327.01	66	3.0061	3.1199	5	Spline PW-CHI	1	-5225.63	121	2.9803	3.1889
6	AN	0	-5329.86	66	3.0077	3.1215	6	AN	1	-5220.67	121	2.9775	3.1861
6	Poly	0	-5327.10	66	3.0062	3.1200	6	Poly	1	-5220.11	121	2.9772	3.1858
6	Step	0	-5337.05	55	3.0056	3.1004	6	Step	1	-5261.23	110	2.9940	3.1836
6	Spline L	0	-5317.97	77	3.0072	3.1400	6	Spline L	1	-5210.75	132	2.9781	3.2057
6	Spline C	0	-5315.56	77	3.0059	3.1386	6	Spline C	1	-5214.35	132	2.9801	3.2077
6	Spline PW-C	0	-5311.84	77	3.0038	3.1366	6	Spline PW-C	1	-5218.08	132	2.9822	3.2098
6	Spline PW-CHI	0	-5315.42	77	3.0058	3.1385	6	Spline PW-CHI	1	-5212.88	132	2.9793	3.2069
7	AN	0	-5316.61	77	3.0065	3.1392	7	AN	1	-5219.76	132	2.9831	3.2107
7	Poly	0	-5310.16	77	3.0029	3.1356	7	Poly	1	-5206.29	132	2.9756	3.2032
7	Step	0	-5326.88	66	3.0061	3.1198	7	Step	1	-5241.47	121	2.9891	3.1977
7	Spline L	0	-5295.21	88	3.0007	3.1524	7	Spline L	1	-5197.44	143	2.9768	3.2234
7	Spline C	0	-5291.17	88	2.9984	3.1501	7	Spline C	1	-5210.15	143	2.9839	3.2304
7	Spline PW-C	0	-5294.12	88	3.0001	3.1518	7	Spline PW-C	1	-5213.05	143	2.9855	3.2321
7	Spline PW-CHI	0	-5291.87	88	2.9988	3.1505	7	Spline PW-CHI	1	-5193.31	143	2.9745	3.2210
8	AN	0	-5297.63	88	3.0020	3.1537	8	AN	1	-5219.19	143	2.9890	3.2355
8	Poly	0	-5295.12	88	3.0006	3.1523	8	Poly	1	-5193.75	143	2.9748	3.2213
8	Step	0	-5316.13	77	3.0062	3.1389	8	Step	1	-5236.19	132	2.9923	3.2199
8	Spline L	0	-5283.98	99	3.0005	3.1712	8	Spline L	1	-5183.82	154	2.9754	3.2409
8	Spline C	0	-5283.35	99	3.0002	3.1709	8	Spline C	1	-5189.82	154	2.9787	3.2442
8	Spline PW-C	0	-5283.11	99	3.0001	3.1707	8	Spline PW-C	1	-5205.32	154	2.9874	3.2528
8	Spline PW-CHI	0	-5284.02	99	3.0006	3.1712	8	Spline PW-CHI	1	-5184.27	154	2.9756	3.2411
9	AN	0	-5296.86	99	3.0077	3.1784	9	AN	1	-5218.32	154	2.9946	3.2601
9	Poly	0	-5291.34	99	3.0046	3.1753	9	Poly	1	-5192.17	154	2.9800	3.2455
9	Step	0	-5309.10	88	3.0084	3.1601	9	Step	1	-5207.53	143	2.9825	3.2290
9	Spline L	0	-5266.48	110	2.9969	3.1866	9	Spline L	1	-5176.29	165	2.9773	3.2618
9	Spline C	0	-5263.48	110	2.9952	3.1849	9	Spline C	1	-5186.27	165	2.9829	3.2673
9	Spline PW-C	0	-5266.26	110	2.9968	3.1864	9	Spline PW-C	1	-5197.10	165	2.9889	3.2733
9	Spline PW-CHI	0	-5264.37	110	2.9957	3.1854	9	Spline PW-CHI	1	-5169.05	165	2.9733	3.2577
10	AN	0	-5294.91	110	3.0128	3.2024	10	AN	1	-5215.21	165	2.9990	3.2834
10	Poly	0	-5274.65	110	3.0015	3.1911	10	Poly	1	-5181.69	165	2.9803	3.2648
10	Step	0	-5297.19	99	3.0079	3.1786	10	Step	1	-5190.75	154	2.9792	3.2447
10	Spline L	0	-5252.42	121	2.9952	3.2038	10	Spline L	1	-5155.31	176	2.9717	3.2752
10	Spline C	0	-5252.20	121	2.9951	3.2037	10	Spline C	1	-5168.39	176	2.9790	3.2824
10	Spline PW-C	0	-5258.10	121	2.9984	3.2070	10	Spline PW-C	1	-5196.28	176	2.9946	3.2980
10	Spline PW-CHI	0	-5248.15	121	2.9928	3.2014	10	Spline PW-CHI	1	-5159.90	176	2.9743	3.2777

Achtnich (2012)

Order	Dist	FullCov	LL	param	AIC/n	BIC/n	Order	Dist	FullCov	LL	param	AIC/n	BIC/n
MNL	replicated		-6095.39	15	3.4060	3.4319							
MNL			-6115.61	11	3.4151	3.4340							
MXL_d	replicated	0	-5280.18	25	2.9572	3.0003	MXL	replicated	1	-5107.91	70	2.8862	3.0069
MXL_d	n	0	-5332.96	22	2.9849	3.0229	MXL	n	1	-5166.28	77	2.9227	3.0554
MXL_d	n + l	0	-5274.34	22	2.9522	2.9902	MXL	n + l	1	-5066.48	77	2.8670	2.9998
2	AN	0	-5376.59	22	3.0092	3.0472	2	AN	1	-5265.42	77	2.9779	3.1107
2	Poly	0	-5376.59	22	3.0092	3.0472	2	Poly	1	-5265.42	77	2.9779	3.1107
2	Step	0	-5396.14	11	3.0140	3.0330	2	Step	1	-5345.97	66	3.0167	3.1305
2	Spline L	0	-5367.68	33	3.0104	3.0673	2	Spline L	1	-5258.81	88	2.9804	3.1321
2	Spline C	0	-5367.37	33	3.0102	3.0671	2	Spline C	1	-5267.12	88	2.9850	3.1367
2	Spline PW-C	0	-5362.78	33	3.0077	3.0646	2	Spline PW-C	1	-5283.65	88	2.9942	3.1459
2	Spline PW-CHI	0	-5368.87	33	3.0111	3.0680	2	Spline PW-CHI	1	-5270.86	88	2.9871	3.1388

Bergmann et al. (2008)

Order	Dist	FullCov	LL	param.	AIC/n	BIC/n
MNL	replicated		-495.59	12	1.2261	1.3182
MNL			-503.53	9	1.2380	1.2893
MXL_d	replicated	0	-481.89	19	1.2099	1.3182
MXL_d	n	0	-487.26	18	1.2204	1.3230
MXL_d	n + l	0	-484.80	18	1.2145	1.3171
2	AN	0	-471.98	18	1.1835	1.2861
2	Poly	0	-471.98	18	1.1835	1.2861
2	Step	0	-486.95	9	1.1980	1.2493
2	Spline L	0	-445.95	27	1.1424	1.2963
2	Spline C	0	-442.63	27	1.1344	1.2882
2	Spline PW-C	0	-451.81	27	1.1565	1.3104
2	Spline PW-CHI	0	-431.83	27	1.1083	1.2622
3	AN	0	-451.88	27	1.1567	1.3106
3	Poly	0	-451.88	27	1.1567	1.3106
3	Step	0	-478.31	18	1.1988	1.3014
3	Spline L	0	-446.67	36	1.1659	1.3710
3	Spline C	0	-426.22	36	1.1165	1.3216
3	Spline PW-C	0	-428.17	36	1.1212	1.3264
3	Spline PW-CHI	0	-446.18	36	1.1647	1.3699
4	AN	0	-445.11	36	1.1621	1.3673
4	Poly	0	-445.11	36	1.1621	1.3673
4	Step	0	-460.71	27	1.1780	1.3319
4	Spline L	0	-428.56	45	1.1439	1.4003
4	Spline C	0	-404.04	45	1.0846	1.3411
4	Spline PW-C	0	-409.79	45	1.0985	1.3550
4	Spline PW-CHI	0	-395.41	45	1.0638	1.3203
5	AN	0	-441.35	45	1.1748	1.4312
5	Poly	0	-441.80	45	1.1758	1.4323
5	Step	0	-454.93	36	1.1858	1.3910
5	Spline L	0	-389.00	54	1.0700	1.3778
5	Spline C	0	-385.59	54	1.0618	1.3696
5	Spline PW-C	0	-384.70	54	1.0597	1.3674
5	Spline PW-CHI	0	-388.06	54	1.0678	1.3756
6	AN	0	-427.63	54	1.1634	1.4711
6	Poly	0	-407.98	54	1.1159	1.4237
6	Step	0	-442.91	45	1.1785	1.4350
6	Spline L	0	-377.37	63	1.0637	1.4227
6	Spline C	0	-370.07	63	1.0461	1.4051
6	Spline PW-C	0	-381.76	63	1.0743	1.4334
6	Spline PW-CHI	0	-385.96	63	1.0844	1.4435
7	AN	0	-393.39	63	1.1024	1.4614
7	Poly	0	-407.98	63	1.1376	1.4967
7	Step	0	-428.61	54	1.1657	1.4735
7	Spline L	0	-377.87	72	1.0866	1.4970
7	Spline C	0	-361.62	72	1.0474	1.4577
7	Spline PW-C	0	-371.96	72	1.0724	1.4827
7	Spline PW-CHI	0	-384.54	72	1.1027	1.5131
8	AN	0	-393.39	72	1.1241	1.5345
8	Poly	0	-407.98	72	1.1594	1.5697
8	Step	0	-389.07	63	1.0919	1.4510
8	Spline L	0	-357.37	81	1.0589	1.5205
8	Spline C	0	-353.66	81	1.0499	1.5115
8	Spline PW-C	0	-348.84	81	1.0383	1.4999
8	Spline PW-CHI	0	-360.17	81	1.0656	1.5273
9	AN	0	-393.39	81	1.1459	1.6075
9	Poly	0	-407.98	81	1.1811	1.6428
9	Step	0	-377.50	72	1.0857	1.4961
9	Spline L	0	-345.15	90	1.0511	1.5640
9	Spline C	0	-349.06	90	1.0605	1.5735
9	Spline PW-C	0	-360.47	90	1.0881	1.6010
9	Spline PW-CHI	0	-350.13	90	1.0631	1.5761
10	AN	0	-393.39	90	1.1676	1.6805
10	Poly	0	-407.98	90	1.2029	1.7158
10	Step	0	-360.59	81	1.0666	1.5283
10	Spline L	0	-344.39	99	1.0710	1.6352
10	Spline C	0	-344.56	99	1.0714	1.6356
10	Spline PW-C	0	-349.22	99	1.0827	1.6469
10	Spline PW-CHI	0	-347.00	99	1.0773	1.6415

Carlsson et al. (2008)

Order	Dist	FullCov	LL	param.	AIC/n	BIC/n	Order	Dist	FullCov	LL	param.	AIC/n	BIC/n
MNL	replicated		-2522.50	73	1.0192	1.1129							
MNL			-2559.44	13	1.0102	1.0269							
MXL_d	replicated	0	-2471.99	85	1.0041	1.1132	MXL	replicated	1	-2309.57	151	0.9663	1.1600
MXL_d	n	0	-2229.20	26	0.8856	0.9190	MXL	n	1	-2089.00	104	0.8612	0.9946
MXL_d	n + l	0	-2110.58	26	0.8390	0.8724	MXL	n + l	1	-1955.40	104	0.8087	0.9422
2	AN	0	-2205.46	26	0.8763	0.9096	2	AN	1	-2027.51	104	0.8370	0.9705
2	Poly	0	-2205.46	26	0.8763	0.9096	2	Poly	1	-2033.94	104	0.8396	0.9730
2	Step	0	-2225.83	13	0.8792	0.8959	2	Step	1	-2069.32	91	0.8483	0.9651
2	Spline L	0	-2116.09	39	0.8463	0.8963	2	Spline L	1	-1963.99	117	0.8172	0.9673
2	Spline C	0	-2117.48	39	0.8468	0.8969	2	Spline C	1	-1958.86	117	0.8152	0.9653
2	Spline PW-C	0	-2121.66	39	0.8485	0.8985	2	Spline PW-C	1	-1972.40	117	0.8205	0.9706
2	Spline PW-CHI	0	-2119.73	39	0.8477	0.8978	2	Spline PW-CHI	1	-1981.42	117	0.8240	0.9742
3	AN	0	-2117.49	39	0.8468	0.8969	3	AN	1	-1969.44	117	0.8193	0.9695
3	Poly	0	-2117.49	39	0.8468	0.8969	3	Poly	1	-1973.36	117	0.8209	0.9710
3	Step	0	-2216.63	26	0.8807	0.9140	3	Step	1	-2029.68	104	0.8379	0.9713
3	Spline L	0	-2104.75	52	0.8469	0.9137	3	Spline L	1	-1926.31	130	0.8075	0.9743
3	Spline C	0	-2107.57	52	0.8481	0.9148	3	Spline C	1	-1949.04	130	0.8164	0.9833
3	Spline PW-C	0	-2108.54	52	0.8484	0.9152	3	Spline PW-C	1	-1948.04	130	0.8160	0.9829
3	Spline PW-CHI	0	-2099.59	52	0.8449	0.9117	3	Spline PW-CHI	1	-1939.50	130	0.8127	0.9795
4	AN	0	-2105.25	52	0.8471	0.9139	4	AN	1	-1950.98	130	0.8172	0.9840
4	Poly	0	-2105.25	52	0.8471	0.9139	4	Poly	1	-1955.83	130	0.8191	0.9859
4	Step	0	-2149.62	39	0.8595	0.9095	4	Step	1	-2013.18	117	0.8365	0.9867
4	Spline L	0	-2088.44	65	0.8456	0.9291	4	Spline L	1	-1913.64	143	0.8076	0.9911
4	Spline C	0	-2084.73	65	0.8442	0.9276	4	Spline C	1	-1923.72	143	0.8116	0.9951
4	Spline PW-C	0	-2083.86	65	0.8438	0.9273	4	Spline PW-C	1	-1914.37	143	0.8079	0.9914
4	Spline PW-CHI	0	-2079.61	65	0.8422	0.9256	4	Spline PW-CHI	1	-1915.14	143	0.8082	0.9917
5	AN	0	-2099.15	65	0.8499	0.9333	5	AN	1	-1934.66	143	0.8159	0.9994
5	Poly	0	-2092.46	65	0.8472	0.9306	5	Poly	1	-1948.54	143	0.8213	1.0048
5	Step	0	-2100.50	52	0.8453	0.9120	5	Step	1	-1968.40	130	0.8240	0.9909
5	Spline L	0	-2083.44	78	0.8488	0.9489	5	Spline L	1	-1904.19	156	0.8090	1.0092
5	Spline C	0	-2072.46	78	0.8445	0.9446	5	Spline C	1	-1898.31	156	0.8067	1.0069
5	Spline PW-C	0	-2074.68	78	0.8453	0.9454	5	Spline PW-C	1	-1904.63	156	0.8092	1.0094
5	Spline PW-CHI	0	-2066.65	78	0.8422	0.9423	5	Spline PW-CHI	1	-1901.84	156	0.8081	1.0083
6	AN	0	-2080.90	78	0.8478	0.9479	6	AN	1	-1929.60	156	0.8190	1.0192
6	Poly	0	-2080.71	78	0.8477	0.9478	6	Poly	1	-1941.99	156	0.8239	1.0241
6	Step	0	-2096.11	65	0.8487	0.9321	6	Step	1	-1955.63	143	0.8241	1.0076
6	Spline L	0	-2056.59	91	0.8433	0.9601	6	Spline L	1	-1872.02	169	0.8015	1.0184
6	Spline C	0	-2058.81	91	0.8442	0.9610	6	Spline C	1	-1897.49	169	0.8115	1.0284
6	Spline PW-C	0	-2059.45	91	0.8445	0.9613	6	Spline PW-C	1	-1879.07	169	0.8043	1.0211
6	Spline PW-CHI	0	-2053.45	91	0.8421	0.9589	6	Spline PW-CHI	1	-1891.70	169	0.8092	1.0261
7	AN	0	-2073.03	91	0.8498	0.9666	7	AN	1	-1925.03	169	0.8223	1.0392
7	Poly	0	-2055.35	91	0.8429	0.9596	7	Poly	1	-1929.16	169	0.8239	1.0408
7	Step	0	-2072.82	78	0.8446	0.9447	7	Step	1	-1927.78	156	0.8183	1.0185
7	Spline L	0	-2029.92	104	0.8380	0.9714	7	Spline L	1	-1847.56	182	0.7970	1.0306
7	Spline C	0	-2033.13	104	0.8392	0.9727	7	Spline C	1	-1790.26	182	0.7745	1.0081
7	Spline PW-C	0	-2026.54	104	0.8367	0.9701	7	Spline PW-C	1	-1863.36	182	0.8032	1.0368
7	Spline PW-CHI	0	-2034.84	104	0.8399	0.9734	7	Spline PW-CHI	1	-1767.48	182	0.7656	0.9991
8	AN	0	-2067.20	104	0.8526	0.9861	8	AN	1	-1920.15	182	0.8255	1.0591
8	Poly	0	-2048.26	104	0.8452	0.9786	8	Poly	1	-1911.30	182	0.8220	1.0556
8	Step	0	-2060.87	91	0.8450	0.9618	8	Step	1	-1908.44	169	0.8158	1.0327
8	Spline L	0	-2012.09	117	0.8361	0.9862	8	Spline L	1	-1843.74	195	0.8006	1.0508
8	Spline C	0	-2001.43	117	0.8319	0.9820	8	Spline C	1	-1758.97	195	0.7673	1.0176
8	Spline PW-C	0	-2016.55	117	0.8378	0.9880	8	Spline PW-C	1	-1788.05	195	0.7787	1.0290
8	Spline PW-CHI	0	-2009.91	117	0.8352	0.9854	8	Spline PW-CHI	1	-1769.44	195	0.7714	1.0217
9	AN	0	-2060.12	117	0.8549	1.0051	9	AN	1	-1911.26	195	0.8271	1.0774
9	Poly	0	-2043.35	117	0.8484	0.9985	9	Poly	1	-1887.16	195	0.8177	1.0679
9	Step	0	-2048.96	104	0.8455	0.9789	9	Step	1	-1898.46	182	0.8170	1.0505
9	Spline L	0	-1972.70	130	0.8257	0.9925	9	Spline L	1	-1720.14	208	0.7572	1.0241
9	Spline C	0	-1996.62	130	0.8351	1.0019	9	Spline C	1	-1696.32	208	0.7478	1.0147
9	Spline PW-C	0	-1999.72	130	0.8363	1.0032	9	Spline PW-C	1	-1693.19	208	0.7466	1.0135
9	Spline PW-CHI	0	-1999.65	130	0.8363	1.0031	9	Spline PW-CHI	1	-1708.72	208	0.7527	1.0196
10	AN	0	-2052.16	130	0.8569	1.0237	10	AN	1	-1907.55	208	0.8308	1.0977
10	Poly	0	-2036.90	130	0.8509	1.0178	10	Poly	1	-1883.48	208	0.8213	1.0882
10	Step	0	-2015.15	117	0.8373	0.9874	10	Step	1	-1842.92	195	0.8003	1.0505
10	Spline L	0	-1985.99	143	0.8360	1.0196	10	Spline L	1	-1680.79	221	0.7468	1.0304
10	Spline C	0	-1980.45	143	0.8339	1.0174	10	Spline C	1	-1694.60	221	0.7523	1.0358
10	Spline PW-C	0	-1981.76	143	0.8344	1.0179	10	Spline PW-C	1	-1710.45	221	0.7585	1.0421
10	Spline PW-CHI	0	-1960.92	143	0.8262	1.0097	10	Spline PW-CHI	1	-1669.59	221	0.7424	1.0260

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Order	Dist	FullCov	LL	param.	AIC/n	BIC/n	Order	Dist	FullCov	LL	param.	AIC/n	BIC/n
MNL	replicated		-13372.37	19	1.0403	1.0463							
MNL			-13514.07	12	1.0507	1.0545							
MXL_d	replicated	0	-11950.56	23	0.9301	0.9374	MXL	replicated	1	-11750.08	29	0.9150	0.9242
MXL_d	n	0	-11354.00	24	0.8839	0.8915	MXL	n	1	-11058.92	90	0.8661	0.8946
MXL_d	n + l	0	-11270.22	24	0.8774	0.8850	MXL	n + l	1	-10902.01	90	0.8539	0.8824
2	AN	0	-11363.48	24	0.8846	0.8922	2	AN	1	-11153.33	90	0.8734	0.9019
2	Poly	0	-11363.48	24	0.8846	0.8922	2	Poly	1	-11153.33	90	0.8734	0.9019
2	Step	0	-11390.09	12	0.8857	0.8895	2	Step	1	-11232.67	78	0.8786	0.9033
2	Spline L	0	-11310.48	36	0.8814	0.8928	2	Spline L	1	-11121.32	102	0.8718	0.9042
2	Spline C	0	-11308.48	36	0.8813	0.8927	2	Spline C	1	-11121.93	102	0.8719	0.9042
2	Spline PW-C	0	-11309.69	36	0.8814	0.8928	2	Spline PW-C	1	-11150.62	102	0.8741	0.9064
2	Spline PW-CHI	0	-11310.31	36	0.8814	0.8928	2	Spline PW-CHI	1	-11124.96	102	0.8721	0.9044
3	AN	0	-11303.71	36	0.8809	0.8923	3	AN	1	-11117.33	102	0.8715	0.9039
3	Poly	0	-11303.71	36	0.8809	0.8923	3	Poly	1	-11117.33	102	0.8715	0.9039
3	Step	0	-11358.33	24	0.8842	0.8918	3	Step	1	-11184.52	90	0.8758	0.9043
3	Spline L	0	-11290.34	48	0.8808	0.8960	3	Spline L	1	-11104.85	114	0.8715	0.9076
3	Spline C	0	-11291.54	48	0.8809	0.8961	3	Spline C	1	-11103.52	114	0.8714	0.9075
3	Spline PW-C	0	-11298.42	48	0.8814	0.8966	3	Spline PW-C	1	-11117.43	114	0.8725	0.9086
3	Spline PW-CHI	0	-11288.02	48	0.8806	0.8958	3	Spline PW-CHI	1	-11091.38	114	0.8705	0.9066
4	AN	0	-11287.53	48	0.8806	0.8958	4	AN	1	-11102.24	114	0.8713	0.9074
4	Poly	0	-11287.53	48	0.8806	0.8958	4	Poly	1	-11102.23	114	0.8713	0.9074
4	Step	0	-11332.82	36	0.8832	0.8946	4	Step	1	-11181.57	102	0.8765	0.9088
4	Spline L	0	-11283.99	60	0.8812	0.9002	4	Spline L	1	-11089.77	126	0.8713	0.9112
4	Spline C	0	-11284.27	60	0.8812	0.9003	4	Spline C	1	-11102.14	126	0.8722	0.9121
4	Spline PW-C	0	-11287.47	60	0.8815	0.9005	4	Spline PW-C	1	-11126.82	126	0.8741	0.9141
4	Spline PW-CHI	0	-11283.92	60	0.8812	0.9002	4	Spline PW-CHI	1	-11093.25	126	0.8715	0.9114
5	AN	0	-11280.09	60	0.8809	0.8999	5	AN	1	-11095.13	126	0.8717	0.9116
5	Poly	0	-11280.09	60	0.8809	0.8999	5	Poly	1	-11093.08	126	0.8715	0.9114
5	Step	0	-11301.88	48	0.8817	0.8969	5	Step	1	-11135.22	114	0.8739	0.9100
5	Spline L	0	-11275.10	72	0.8815	0.9043	5	Spline L	1	-11087.05	138	0.8720	0.9157
5	Spline C	0	-11270.07	72	0.8811	0.9039	5	Spline C	1	-11077.03	138	0.8712	0.9149
5	Spline PW-C	0	-11274.41	72	0.8814	0.9042	5	Spline PW-C	1	-11094.12	138	0.8725	0.9163
5	Spline PW-CHI	0	-11269.65	72	0.8810	0.9039	5	Spline PW-CHI	1	-11090.29	138	0.8722	0.9160
6	AN	0	-11262.13	72	0.8805	0.9033	6	AN	1	-11092.59	138	0.8724	0.9161
6	Poly	0	-11262.12	72	0.8805	0.9033	6	Poly	1	-11080.23	138	0.8715	0.9152
6	Step	0	-11297.10	60	0.8822	0.9012	6	Step	1	-11121.32	126	0.8737	0.9136
6	Spline L	0	-11256.76	84	0.8810	0.9076	6	Spline L	1	-11068.02	150	0.8714	0.9190
6	Spline C	0	-11250.86	84	0.8805	0.9071	6	Spline C	1	-11081.47	150	0.8725	0.9200
6	Spline PW-C	0	-11256.18	84	0.8809	0.9075	6	Spline PW-C	1	-11098.74	150	0.8738	0.9213
6	Spline PW-CHI	0	-11254.54	84	0.8808	0.9074	6	Spline PW-CHI	1	-11077.01	150	0.8721	0.9197
7	AN	0	-11257.00	84	0.8810	0.9076	7	AN	1	-11091.48	150	0.8733	0.9208
7	Poly	0	-11256.46	84	0.8809	0.9076	7	Poly	1	-11076.70	150	0.8721	0.9196
7	Step	0	-11277.98	72	0.8817	0.9045	7	Step	1	-11099.85	138	0.8730	0.9167
7	Spline L	0	-11243.84	96	0.8809	0.9113	7	Spline L	1	-11060.04	162	0.8718	0.9231
7	Spline C	0	-11249.48	96	0.8813	0.9118	7	Spline C	1	-11069.18	162	0.8725	0.9238
7	Spline PW-C	0	-11254.07	96	0.8817	0.9121	7	Spline PW-C	1	-11090.15	162	0.8741	0.9254
7	Spline PW-CHI	0	-11243.99	96	0.8809	0.9113	7	Spline PW-CHI	1	-11060.92	162	0.8718	0.9231
8	AN	0	-11255.93	96	0.8818	0.9123	8	AN	1	-11087.25	162	0.8739	0.9252
8	Poly	0	-11249.05	96	0.8813	0.9117	8	Poly	1	-11075.24	162	0.8729	0.9243
8	Step	0	-11266.04	84	0.8817	0.9083	8	Step	1	-11077.36	150	0.8722	0.9197
8	Spline L	0	-11233.88	108	0.8811	0.9153	8	Spline L	1	-11048.49	174	0.8718	0.9269
8	Spline C	0	-11230.83	108	0.8808	0.9150	8	Spline C	1	-11053.93	174	0.8722	0.9273
8	Spline PW-C	0	-11230.91	108	0.8808	0.9150	8	Spline PW-C	1	-11070.14	174	0.8735	0.9286
8	Spline PW-CHI	0	-11229.19	108	0.8807	0.9149	8	Spline PW-CHI	1	-11046.70	174	0.8716	0.9268
9	AN	0	-11249.84	108	0.8823	0.9165	9	AN	1	-11085.83	174	0.8747	0.9298
9	Poly	0	-11246.09	108	0.8820	0.9162	9	Poly	1	-11070.52	174	0.8735	0.9286
9	Step	0	-11255.76	96	0.8818	0.9122	9	Step	1	-11072.78	162	0.8727	0.9241
9	Spline L	0	-11218.31	120	0.8808	0.9188	9	Spline L	1	-11029.48	186	0.8712	0.9302
9	Spline C	0	-11220.80	120	0.8810	0.9190	9	Spline C	1	-11054.24	186	0.8732	0.9321
9	Spline PW-C	0	-11220.82	120	0.8810	0.9190	9	Spline PW-C	1	-11063.94	186	0.8739	0.9328
9	Spline PW-CHI	0	-11219.70	120	0.8809	0.9189	9	Spline PW-CHI	1	-11030.08	186	0.8713	0.9302
10	AN	0	-11249.56	120	0.8832	0.9212	10	AN	1	-11085.08	186	0.8756	0.9345
10	Poly	0	-11234.45	120	0.8820	0.9201	10	Poly	1	-11069.80	186	0.8744	0.9333
10	Step	0	-11234.17	108	0.8811	0.9153	10	Step	1	-11064.29	174	0.8730	0.9281
10	Spline L	0	-11201.21	132	0.8804	0.9222	10	Spline L	1	-11027.38	198	0.8720	0.9347
10	Spline C	0	-11202.16	132	0.8805	0.9223	10	Spline C	1	-11021.56	198	0.8716	0.9343
10	Spline PW-C	0	-11206.52	132	0.8808	0.9226	10	Spline PW-C	1	-11041.25	198	0.8731	0.9358
10	Spline PW-CHI	0	-11199.74	132	0.8803	0.9221	10	Spline PW-CHI	1	-11031.53	198	0.8723	0.9351

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Order	Dist	FullCov	LL	param.	AIC/n	BIC/n	Order	Dist	FullCov	LL	param.	AIC/n	BIC/n
MNL	replicated		-7350.75	9	2.0219	2.0304							
MNL			-7350.75	9	2.0219	2.0304							
MXL_d	replicated	0	-5729.41	11	1.5770	1.5875	MXL	replicated	1	-5479.75	12	1.5087	1.5201
MXL_d	n	0	-5245.83	18	1.4461	1.4631	MXL	n	1	-4630.89	54	1.2871	1.3382
MXL_d	n + l	0	-5246.90	18	1.4464	1.4634	MXL	n + l	1	-4617.92	54	1.2835	1.3346
2	AN	0	-5278.65	18	1.4551	1.4722	2	AN	1	-4911.37	54	1.3641	1.4152
2	Poly	0	-5278.65	18	1.4551	1.4722	2	Poly	1	-4912.68	54	1.3645	1.4156
2	Step	0	-5300.90	9	1.4588	1.4673	2	Step	1	-5087.36	45	1.4100	1.4526
2	Spline L	0	-5255.00	27	1.4511	1.4767	2	Spline L	1	-4925.43	63	1.3704	1.4301
2	Spline C	0	-5249.90	27	1.4497	1.4753	2	Spline C	1	-4905.03	63	1.3648	1.4245
2	Spline PW-C	0	-5248.29	27	1.4493	1.4748	2	Spline PW-C	1	-4907.98	63	1.3657	1.4253
2	Spline PW-CHI	0	-5255.36	27	1.4512	1.4768	2	Spline PW-CHI	1	-4935.68	63	1.3733	1.4329
3	AN	0	-5232.41	27	1.4449	1.4705	3	AN	1	-4880.00	63	1.3580	1.4176
3	Poly	0	-5232.41	27	1.4449	1.4705	3	Poly	1	-4878.19	63	1.3575	1.4171
3	Step	0	-5281.80	18	1.4560	1.4730	3	Step	1	-4992.57	54	1.3864	1.4375
3	Spline L	0	-5214.01	36	1.4423	1.4764	3	Spline L	1	-4899.00	72	1.3657	1.4338
3	Spline C	0	-5213.49	36	1.4422	1.4763	3	Spline C	1	-4915.67	72	1.3702	1.4384
3	Spline PW-C	0	-5217.21	36	1.4432	1.4773	3	Spline PW-C	1	-4932.84	72	1.3750	1.4431
3	Spline PW-CHI	0	-5212.69	36	1.4419	1.4760	3	Spline PW-CHI	1	-4909.04	72	1.3684	1.4366
4	AN	0	-5218.71	36	1.4436	1.4777	4	AN	1	-4869.44	72	1.3575	1.4257
4	Poly	0	-5218.71	36	1.4436	1.4777	4	Poly	1	-4869.36	72	1.3575	1.4257
4	Step	0	-5262.50	27	1.4532	1.4787	4	Step	1	-4983.30	63	1.3863	1.4460
4	Spline L	0	-5212.67	45	1.4444	1.4870	4	Spline L	1	-4889.19	81	1.3654	1.4421
4	Spline C	0	-5209.73	45	1.4436	1.4862	4	Spline C	1	-4894.99	81	1.3670	1.4437
4	Spline PW-C	0	-5206.78	45	1.4428	1.4854	4	Spline PW-C	1	-4907.68	81	1.3705	1.4472
4	Spline PW-CHI	0	-5213.52	45	1.4446	1.4873	4	Spline PW-CHI	1	-4893.94	81	1.3667	1.4434
5	AN	0	-5211.44	45	1.4441	1.4867	5	AN	1	-4856.00	81	1.3563	1.4330
5	Poly	0	-5211.42	45	1.4441	1.4867	5	Poly	1	-4855.83	81	1.3563	1.4330
5	Step	0	-5239.28	36	1.4493	1.4833	5	Step	1	-4953.33	72	1.3806	1.4488
5	Spline L	0	-5200.97	54	1.4437	1.4948	5	Spline L	1	-4873.74	90	1.3637	1.4489
5	Spline C	0	-5199.64	54	1.4433	1.4944	5	Spline C	1	-4874.35	90	1.3638	1.4490
5	Spline PW-C	0	-5203.07	54	1.4443	1.4954	5	Spline PW-C	1	-4890.54	90	1.3683	1.4535
5	Spline PW-CHI	0	-5199.33	54	1.4432	1.4944	5	Spline PW-CHI	1	-4876.45	90	1.3644	1.4496
6	AN	0	-5197.48	54	1.4427	1.4938	6	AN	1	-4839.97	90	1.3544	1.4396
6	Poly	0	-5197.44	54	1.4427	1.4938	6	Poly	1	-4839.71	90	1.3543	1.4395
6	Step	0	-5242.07	45	1.4525	1.4951	6	Step	1	-4932.31	81	1.3773	1.4540
6	Spline L	0	-5185.64	63	1.4419	1.5016	6	Spline L	1	-4862.95	99	1.3632	1.4569
6	Spline C	0	-5187.94	63	1.4426	1.5022	6	Spline C	1	-4870.15	99	1.3651	1.4589
6	Spline PW-C	0	-5195.45	63	1.4446	1.5043	6	Spline PW-C	1	-4890.59	99	1.3708	1.4645
6	Spline PW-CHI	0	-5187.30	63	1.4424	1.5020	6	Spline PW-CHI	1	-4865.65	99	1.3639	1.4576
7	AN	0	-5192.35	63	1.4438	1.5034	7	AN	1	-4825.82	99	1.3530	1.4467
7	Poly	0	-5192.36	63	1.4438	1.5034	7	Poly	1	-4817.76	99	1.3508	1.4445
7	Step	0	-5210.24	54	1.4462	1.4973	7	Step	1	-4896.25	90	1.3698	1.4551
7	Spline L	0	-5176.57	72	1.4419	1.5101	7	Spline L	1	-4835.57	108	1.3581	1.4604
7	Spline C	0	-5177.09	72	1.4421	1.5102	7	Spline C	1	-4847.36	108	1.3614	1.4636
7	Spline PW-C	0	-5184.26	72	1.4440	1.5122	7	Spline PW-C	1	-4867.09	108	1.3668	1.4690
7	Spline PW-CHI	0	-5173.32	72	1.4410	1.5092	7	Spline PW-CHI	1	-4837.92	108	1.3588	1.4610
8	AN	0	-5191.07	72	1.4459	1.5141	8	AN	1	-4824.06	108	1.3550	1.4572
8	Poly	0	-5189.44	72	1.4455	1.5136	8	Poly	1	-4813.45	108	1.3520	1.4543
8	Step	0	-5201.20	63	1.4462	1.5059	8	Step	1	-4883.20	99	1.3687	1.4625
8	Spline L	0	-5166.87	81	1.4417	1.5184	8	Spline L	1	-4833.06	117	1.3599	1.4707
8	Spline C	0	-5161.46	81	1.4402	1.5169	8	Spline C	1	-4833.71	117	1.3601	1.4709
8	Spline PW-C	0	-5162.73	81	1.4406	1.5173	8	Spline PW-C	1	-4858.94	117	1.3670	1.4778
8	Spline PW-CHI	0	-5162.34	81	1.4405	1.5172	8	Spline PW-CHI	1	-4819.10	117	1.3561	1.4669
9	AN	0	-5188.10	81	1.4476	1.5242	9	AN	1	-4822.88	117	1.3571	1.4679
9	Poly	0	-5181.40	81	1.4457	1.5224	9	Poly	1	-4812.93	117	1.3544	1.4652
9	Step	0	-5187.63	72	1.4450	1.5131	9	Step	1	-4881.42	108	1.3707	1.4730
9	Spline L	0	-5148.02	90	1.4390	1.5242	9	Spline L	1	-4816.56	126	1.3578	1.4771
9	Spline C	0	-5152.72	90	1.4403	1.5255	9	Spline C	1	-4841.07	126	1.3646	1.4839
9	Spline PW-C	0	-5158.71	90	1.4420	1.5272	9	Spline PW-C	1	-4836.58	126	1.3633	1.4826
9	Spline PW-CHI	0	-5143.26	90	1.4377	1.5229	9	Spline PW-CHI	1	-4812.81	126	1.3568	1.4761
10	AN	0	-5182.36	90	1.4484	1.5337	10	AN	1	-4822.24	126	1.3594	1.4787
10	Poly	0	-5178.35	90	1.4474	1.5326	10	Poly	1	-4810.38	126	1.3561	1.4754
10	Step	0	-5177.83	81	1.4447	1.5214	10	Step	1	-4861.87	117	1.3678	1.4786
10	Spline L	0	-5147.05	99	1.4412	1.5350	10	Spline L	1	-4802.97	135	1.3566	1.4844
10	Spline C	0	-5145.31	99	1.4407	1.5345	10	Spline C	1	-4812.64	135	1.3592	1.4871
10	Spline PW-C	0	-5141.12	99	1.4396	1.5333	10	Spline PW-C	1	-4817.53	135	1.3606	1.4884
10	Spline PW-CHI	0	-5144.42	99	1.4405	1.5342	10	Spline PW-CHI	1	-4812.27	135	1.3591	1.4870

Robustness to reanalysis – summary

			LL	params.	AIC/n	BIC/n
Achtsnicht (2012)	replication		-5280.18	25	2.96	3.00
	best AIC	MXL_d	-5066.48	77	2.87	3.00
	best BIC	MXL_d	-5274.34	22	2.95	2.99
Bergmann et al. (2008)	replication		-481.89	19	1.21	1.32
	best AIC	Spline C (6)	-348.84	81	1.04	1.50
	best BIC	LML Step (2)	-486.95	9	1.20	1.25
Carlsson et al. (2008)	replication		-2471.99	85	1.00	1.11
	best AIC	Spline PW-CHI (10)	-1669.59	221	0.74	1.03
	best BIC	MXL_d	-2110.58	26	0.84	0.87
Mabit et al. (2011)	replication		-11750.08	29	0.92	0.92
	best AIC	MXL	-10902.01	90	0.85	0.88
	best BIC	MXL	-10902.01	90	0.85	0.88
Scarpa et al. (2010)	replication		-5729.41	11	1.58	1.59
	best AIC	MXL	-4617.92	54	1.28	1.33
	best BIC	MXL	-4617.92	54	1.28	1.33

Reanalysis – even larger WTP differences

E.g., Scarpa et al. (2010) use WTP-space model

	Replicated		Best	
	mean	s.d.	mean	s.d.
Solar electricity	2.93*** (0.16)		7.46*** (0.42)	12.38*** (0.67)
Solar hot water	3.02*** (0.16)		7.47*** (0.41)	11.99*** (0.69)
Wind turbine	2.48*** (0.14)		5.69*** (0.41)	11.82*** (0.65)
Friend	-0.21*** (0.05)		-0.20* (0.11)	0.82*** (0.16)
Heating engineer	-0.04 (0.08)		0.42*** (0.14)	0.96*** (0.18)
Both	0.36*** (0.07)		0.56*** (0.13)	1.07*** (0.17)
Maintenance cost	-0.05*** (0.01)		-0.08*** (0.01)	0.14*** (0.02)
Energy savings	0.01 (0.05)	0.02*** (0.00)	0.29*** (0.05)	0.68*** (0.05)

Summary and conclusions

- Relatively low access to data, codes, documentation (33%)
- The results of the MXL models could not be closely replicated ($\Delta\beta = 1$ to 15%)
 - Simulation error
 - Convergence problems
 - Possibly other
- The results were generally not robust to reanalysis
 - More flexible MXL specification enough to outperform the models reported in the papers
 - In two cases (relatively small datasets) LML performed better than MXL
 - Estimating LML models actually needed a lot more time than expected, due to arbitrariness in the assumptions necessary for model specification (bounds, mixing functions, starting values)
- Generally, the results provide an insight into model uncertainty
 - Uncertainty much larger than the reported standard errors
- More transparency and attention to documenting robustness of one's analysis needed

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